GCE

Further Mathematics A

Y535/01: Additional pure mathematics

AS Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f”.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance
1	(a)	$205 = 7 \times 29 + 2$ i.e. $q = 29$, $r = 2$	B1 [1]	1.1	
1	(b)	From **(a)**, since 7 does not divide exactly into 205 (and 7 is prime), 7 must divide 8666 If 7 | (205×8066), then, by *Euclid's Lemma* …	M1 A1 [2]	2.4 2.2a	Use of *Euclid's Lemma* No need to note that this is because $r \neq 0$ Allow description of the "*Euclid's Lemma*" condition instead A complete justification with assumption and conclusion (hcf(7, 205) = 1)

Question		Answer	Marks	AO	Guidance
2	(a)	$u_{n+5} = = 3(n+5)^2 + 3(n+5) + 7 = 3n^2 + 33n + 97 =$ $= 3n^2 + 3n + 7 \pmod{10} \equiv u_n \pmod{10}$	M1 A1 [2]	3.1a 1.1	Expanding and considering at least one term mod 10 or evaluating u_n from n=1 to n=10 Correct conclusion from correct algebraic work
2	(b)	The sequence is *periodic* (with period 5) Any shorter periodicity would have to be a factor of 5. Since the sequence is not constant, there are no smaller possibilities.	B1 B1 [2]	1.1 2.4	Allow *repeating/cyclic* Allow statement only that sequence is non-constant or sight of {3, 5, (3, 7, 7, …)}.

Question		Answer	Marks	AO	Guidance
3		$\frac{\partial z}{\partial x} = 2xy^2 - 3y + 2$	B1	1.1	
		$\frac{\partial z}{\partial y} = 2x^2 y - 3x + 1$	B1	1.1	
		Setting both first partial derivatives equal to zero *and* attempt to eliminate one variable	M1	1.1a	Either directly via $x = \frac{3y-2}{2y^2}$ or $y = \frac{3x-1}{2x^2}$ **OR** indirectly via $2xy = \frac{3y-2}{y} = \frac{3x-1}{x} \Rightarrow y = 2x$
		Either $y^3 - 3y + 2 = 0$ or $4x^3 - 3x + 1 = 0$ i.e. $(y-1)^2(y+2) = 0$ or $(2x-1)^2(x+1) = 0$	M1	1.1	Any cubic equation in one variable
		$(x, y, z) = (\frac{1}{2}, 1, \frac{3}{4})$	A1	1.1	First SP correct BC www
		$= (-1, -2, -6)$	A1	1.1	Second SP correct BC www **SC1** for both pairs of (x, y) correct with z's missing or z incorrect NB Extra SP A1A0
			[6]		

Question		Answer	Marks	AO	Guidance
4	(a)	**b** and **d** are the direction vectors of l …	B1	1.2	
		… so $\mathbf{b} = m\mathbf{d}$ for some scalar m or $\mathbf{b} \times \mathbf{d} = \mathbf{0}$	B1	2.2a	or statement that one of **b**, **d** is a multiple of the other or that they are parallel
		a and **c** are (the position vectors of) points on l …	B1	1.2	
		$\mathbf{a} - \mathbf{c} = \mu\mathbf{b}$ or $\mathbf{a} - \mathbf{c} = \gamma\mathbf{d}$ or $(\mathbf{a} - \mathbf{c}) \times \mathbf{b} = \mathbf{0}$ or $(\mathbf{a} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$	B1	2.4	or $(\mathbf{a} - \mathbf{c})$ is a multiple of (or parallel to) **b** (or **d**) **SC1** $(\mathbf{a} + \lambda\,\mathbf{b} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$
			[4]		
4	(b)	$(\mathbf{a} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$ since **d**, $\mathbf{a} - \mathbf{c}$ parallel (from **(a)**)	M1	1.1	
		$\Rightarrow \mathbf{a} \times \mathbf{d} = \mathbf{c} \times \mathbf{d}$	A1	1.1	
		$\Rightarrow \mathbf{a} \cdot (\mathbf{c} \times \mathbf{d}) = \mathbf{a} \cdot (\mathbf{a} \times \mathbf{d}) = \mathbf{0}$ since $\mathbf{a} \times \mathbf{d}$ is perpendicular to **a**	A1	2.4	Correct answer, fully justified
		ALT. Substitution for **r** and use of the Distributive property of the VP: $(\mathbf{a} \times \mathbf{d}) + (\lambda\,\mathbf{b} \times \mathbf{d}) - (\mathbf{c} \times \mathbf{d}) = \mathbf{0}$	M1		
		But $\mathbf{b} \times \mathbf{d} = \mathbf{0}$ since **b**, **d** parallel $\Rightarrow \mathbf{a} \times \mathbf{d} = \mathbf{c} \times \mathbf{d}$	A1		
		$\Rightarrow \mathbf{a} \cdot (\mathbf{c} \times \mathbf{d}) = \mathbf{a} \cdot (\mathbf{a} \times \mathbf{d}) = \mathbf{0}$ since $\mathbf{a} \times \mathbf{d}$ is perpendicular to **a**	A1		Correct answer, fully justified
			[3]		

Question			Answer	Marks	AO	Guidance
5	(a)		$7119_{23} = 9 + (1 \times 23) + (1 \times 23^2) + (7 \times 23^3)$ $= 85\,730_{10}$	M1 A1 [2]	1.1 1.1	Clear use of base-23 column-values
5	(b)		$7n + 11 \equiv 9 \pmod{23} \Rightarrow 7n + 11 \equiv 32 \pmod{23}$ **OR** $7n \equiv -2 \pmod{23}$ $\Rightarrow 7n \equiv 21 \pmod{23}$ $\Rightarrow n \equiv 3 \pmod{23}$ or $n = 23k + 3$ ($k \in \mathbb{Z}$) hcf(7, 23) = 1 for division to be valid	M1 A1 B1 [3]	1.1 1.1 2.4	Use of modular arithmetic to gain a multiple of 7 on the RHS Any correct, complete statement Accept $n \equiv -20 \pmod{23}$ Statement that hcf(7, 23) = 1 for division to be valid or equivalent
5	(c)	i	$3N - 7M = 30a + 3b - 7a - 49b = 23(a - 2b)$ If $N = 23k$, then $7M = 23(3k - a + 2b)$ If $M = 23k$, then $3N = 23(7k + a - 2b)$ Both correctly shown multiples of 23 ***with*** explanation that hcf(7, 23) = 1 and hcf(3, 23) = 1	B1 M1 M1 A1 [4]	1.1 2.1 2.1 2.4	Must be written explicitly as a multiple of 23, here or later on Proof attempted in one direction ("proof" includes attempt to obtain a multiple of 23) Proof attempted in other direction ("proof" includes attempt to obtain a multiple of 23) At least one justification must be noted
5	(c)	ii	711 965 → **71 231** → 7130 → 713 → 92 (→ 23) Process may be stopped at either 92 or 23, giving divisibility by 23	M1 A1	1.1 1.1	Implementation of procedure, at least first step correct Conclusion must be noted Accept process stopped at 713 if 23 is shown to be a factor

Question			Answer	Marks	AO	Guidance
				[2]		

Question			Answer	Marks	AO	Guidance
6	(a)		$P_6 = \frac{2\times P_5}{6} + \frac{5}{P_5}$ used with $P_5 = 266.67$ so that $P_6 = 88.91$ to 2d.p.	M1 A1 [1]	1.1 1.1	Must consider a range of possible values of P_6 (ignoring </≤ at endpoints) Shown carefully (showing that P_6 is monotonic in the interval is not required) **SC1** 88.91 (by substituting 266.67 soi)
6	(b)	i	Number of bacteria initially increases, then declines	B1 [1]	2.2a	Must be in context (refers to the number of bacteria)
6	(b)	ii	On the 8th day	B1 [1]	3.4	(∵ the number of bacteria falls below 10 000)
6	(c)	i	$f(n) = \sqrt{n}$	B1 [1]	3.3	Other possibilities for f may also be fine. At least three values need to approximately fit
6	(c)	ii	Suggest $n = 100$ (accept $n = 99$)	B1 [1]	3.1b	**FT** their $f(n)$ even from B0 in part **(c)** provided >80
6	(c)	iii	Using $10.08 = \frac{2x}{100} + \frac{99}{x}$ to create a quadratic eqn. in $x = P_{99}$ which gives $P_{99} = 10.02\ldots$	M1 A1 [2]	3.2a 3.5b	i.e. $2x^2 - 1008x + 9900 = 0$ It is not necessary to verify that $P_{98} < 10$ (working back another step gives $P_{98} = 9.98$)

Question			Answer	Marks	AO	Guidance
6	(d)		$P_{n+1} = \text{INT}\left(\frac{2P_n}{n+1} + \frac{n}{P_n}\right)(+1)$	B1 [1]	3.5c	

Question			Answer	Marks	AO	Guidance
7	(a)		$x = 11$ Since, e.g., $x = 2 \times 16 = 32 \equiv 11 \pmod{21}$ **OR** Calculating the powers of 2, mod 21: 2, 4, 8, 16, 11 …	B1 B1 [2]	3.1a 2.4	 Correct working. **ALT** G consists of all n, $0 < n < 21$, with n co-prime to 21
7	(b)		Element 2 4 5 8 10 13 16 17 19 20 x Order 6 3 6 2 6 2 3 6 6 2 6	B1 B1 B1 [2]	1.1 1.1 1.1	At least 3 correct At least 6 correct All correct
7	(c)	i	Subgroup of order 3 is {1, 4, 16}	B1 [1]	1.1	
7	(c)	ii	Subgroups of order 6 are {1, 2, 4, 8, 16, 11} {1, 5, 4, 20, 16, 17} {1, 19, 4, 13, 16, 10}	B1 B1 B1 [3]	1.1 1.1 1.1	Accept x for 11 Withhold final B1 if extras appear
7	(d)		The order of the elements of the subgroup must divide 4	B1	2.5	Or a group with identity + 3 self-inverse elements

Question			Answer	Marks	AO	Guidance
			A cyclic group of order 4 does not exist since G has no element of order 4	**B1**	**2.1**	Or the only possible order of the elements (other than the identity) is two
			Only subgroup is $\{1, 8, 13, 20\}$	**M1**	**3.1a**	
			Checking for Closure: $8 \times 13 = 20,\quad 8 \times 20 = 13,\quad 13 \times 20 = 8$	**A1**	**1.1**	Visibly checked
				[4]		
7	**(e)**		G has no element of order 12, hence **not** cyclic	**B1**	**2.3**	Correct answer with valid stated reason; no further justification required. (NB This is easily seen from part **(c) (iii)**, where each element appears in a 6-subgroup so cannot possibly generate the whole group).
				[1]		

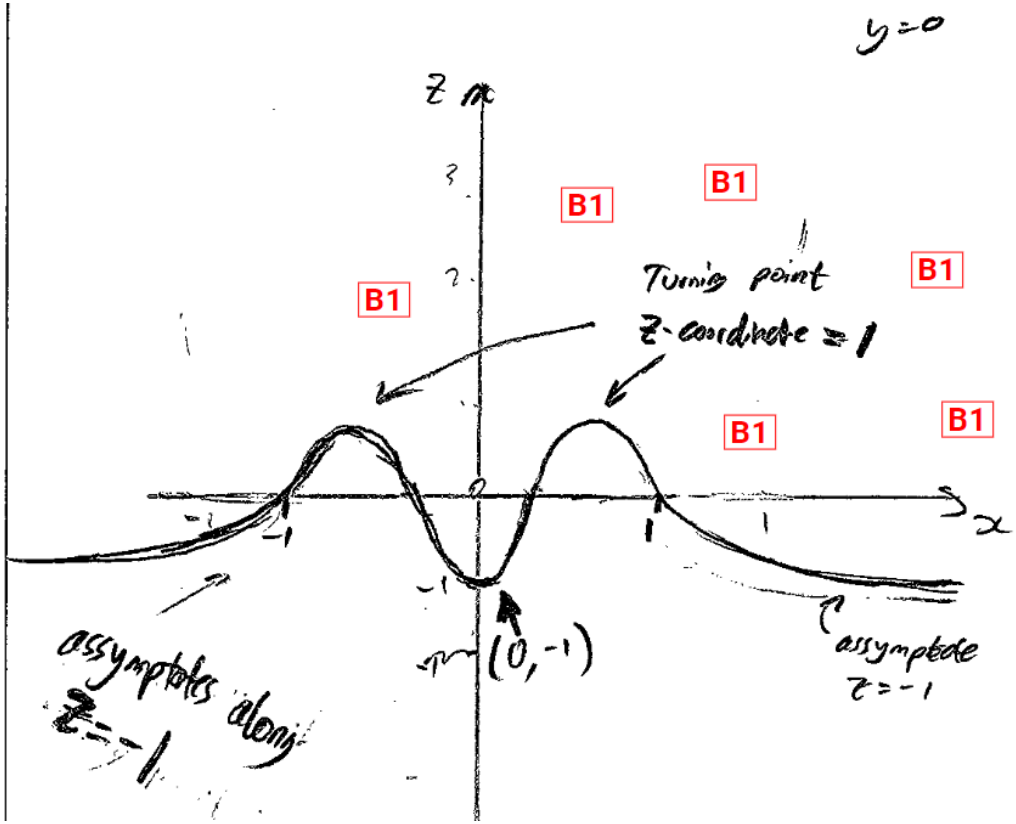
Question		Answer	Marks	AO	Guidance
8		A single, continuous curve drawn in the x-z plane	**B1**	**1.1**	Accept a closed curve
		Curve lies entirely in $-1 \le z \le 1$	**B1**	**1.1**	
		A single point with $z = -1$ at $(0, -1)$	**B1**	**1.1**	
		Two maxima at $(\pm 1, 1)$	**B1**	**1.1**	Maxima do not need to be stationary points
		As $x \to \pm\infty,\ z \to -1$	**B1**	**1.1**	
		An adequately, and essentially completely correct, solution curve drawn	**B1**	**1.1**	**SC1** one of the following in correct location without labels: Curve lies entirely in $-1 \le z \le 1$ A single point with $z = -1$ at $(0, -1)$ Two maxima at $(\pm 1, 1)$

Question		Answer	Marks	AO	Guidance
			[6]		

APPENDIX

Exemplar answers for question 8

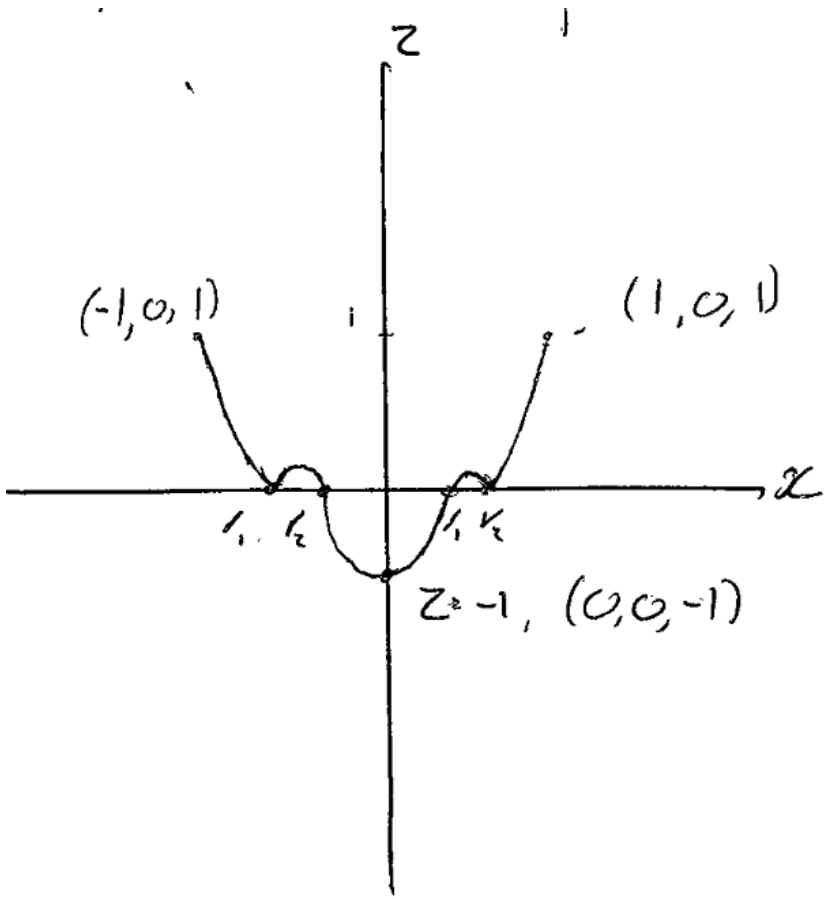
Exemplar 1



As $x \to \pm\infty$, $z \to -1$	**B1**
An adequately, and essentially completely correct, solution curve drawn	**B1**

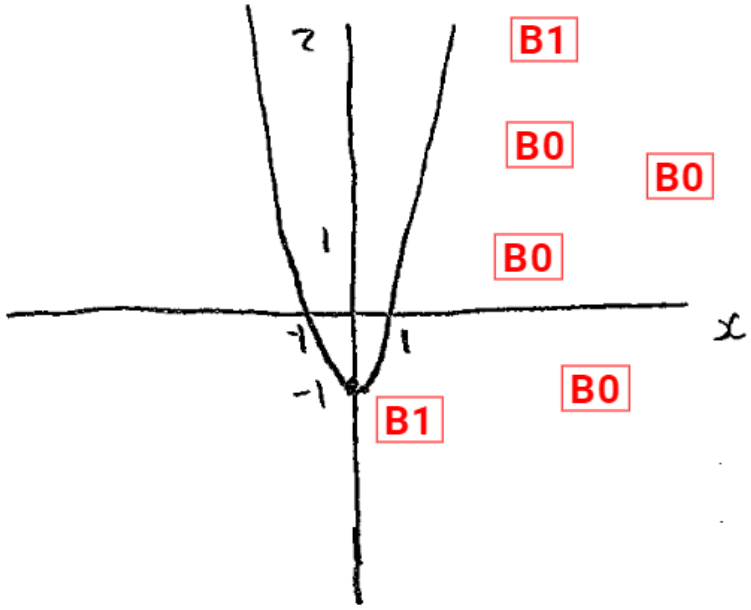
A single, continuous curve drawn in the x-z plane	**B1**
Curve lies entirely in $-1 \le z \le 1$	**B1**
A single point with $z = -1$ at $(0, -1)$	**B1**
Two maxima at $(\pm 1, 1)$	**B1**

Exemplar 2



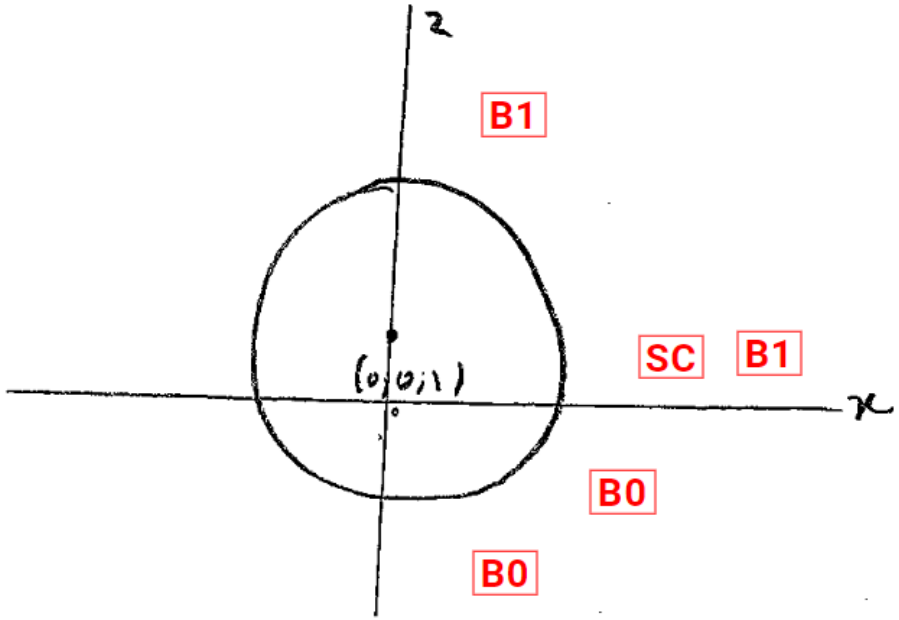
A single, continuous curve drawn in the x-z plane	**B1**
Curve lies entirely in $-1 \le z \le 1$	**B1**
A single point with $z = -1$ at $(0, -1)$	**B1**
Two maxima at $(\pm 1, 1)$	**B1**
As $x \to \pm\infty$, z does not approach -1	**B0**
A partially incorrect solution curve drawn	**B0**

Exemplar 3



A single, continuous curve drawn in the x-z plane	**B1**
Curve does not lie entirely in $-1 \le z \le 1$	**B0**
A single point with $z = -1$ at $(0, -1)$	**B1**
No maxima at $(\pm 1, 1)$	**B0**
As $x \to \pm\infty$, z does not approach -1	**B0**
An incorrect solution curve drawn	**B0**

Exemplar 4



A single, continuous curve drawn in the x-z plane	**B1**
SC1 one of the following in correct location without labels: Curve lies entirely in $-1 \le z \le 1$ **A single point with $z = -1$ at (0, –1)** Two maxima at (±1, 1)	**B1**
As $x \to \pm\infty$, z does not approach –1	**B0**
An incorrect, solution curve drawn	**B0**

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